Name Solutions

November 14, 2012

ECE 311 Exam 3 Fall 2012

Closed Text and Notes

1) Be sure you have X pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) This exam is worth 100 points.

(10 pts) 1. A 1 C charge is moving with a constant velocity of $5\hat{\mathbf{a}}_x \frac{m}{s}$ in a uniform magnetic flux density of $5\hat{\mathbf{a}}_y T$. What must be the electric field intensity?

To be moving with a constant velocity means the total force on the 1 C charge is zero.

$$\vec{F} = q\vec{E} + q\,\vec{u} \times \vec{B} = 0$$

$$\vec{E} = -\vec{u} \times \vec{B} = -5\hat{a}_x \frac{m}{s} \times 5\hat{a}_y \frac{Wb}{m^2}$$

$$= -25\hat{a}_z \frac{Wb}{m\,s}$$

$$= -25\hat{a}_z \frac{V\,s}{m\,s}$$

$$= -25\hat{a}_z \frac{V}{m}$$

(5 pts) 2. A material has a magnetization of $\mathbf{M} = 10\hat{\mathbf{a}}_x \frac{\text{A}}{\text{m}}$. If there are $10^{22} \frac{atoms}{m^3}$, what is the magnetic dipole moment of each atom?

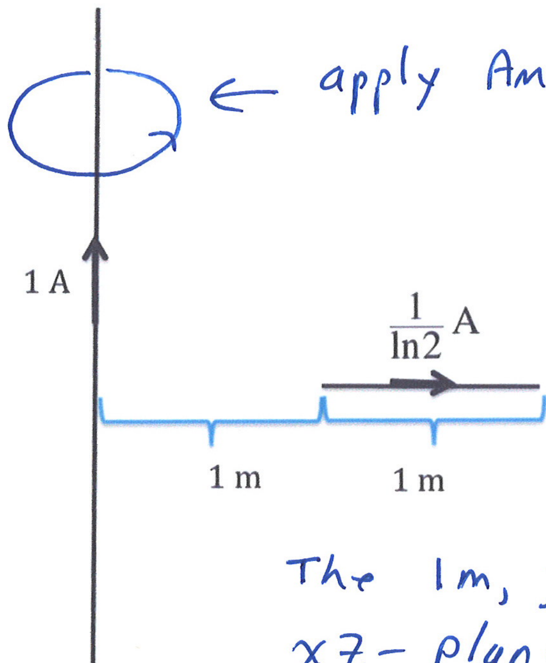
$$\vec{M} = n\vec{p} =$$

$$\vec{p} = \frac{\vec{M}}{n} = \frac{10\hat{a}_x \frac{A}{m}}{10^{22}\ m^{-3}}$$

$$\vec{p} = 10^{-21}\hat{a}_x\ A m^2$$

(12 pts) 3. An infinitely long straight wire lies on the z-axis and has a current flowing through it of 1 A. A 1 m long line with $\frac{1}{\ln 2}$ A flowing through it extends from x = 1 m to x = 2 m as shown. What is the force on the 1 m line? Assume free space everywhere. Note $\mu_o = 4\pi x 10^{-7} \frac{H}{m}$.

First find the magnetic flux density caused by the 1A wire



← apply Ampere's circuit law

$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed} = 1A$$

$$H 2\pi\rho = 1A$$

$$\vec{H} = \frac{1A}{2\pi\rho}\hat{a}_\phi$$

$$\vec{B} = \mu_o \frac{1A}{2\pi\rho}\hat{a}_\phi$$

The 1m, $\frac{1}{\ln 2}$A wire is in the xz-plane. So $\vec{B}$ along this wire can be described as

$$\vec{B} = \mu_o \frac{1A}{2\pi\rho}\hat{a}_y$$

The force on the 1m, $\frac{1}{\ln 2}$ A wire is given by,

$$\vec{F} = \int I\vec{dl} \times \vec{B} = \int_{1m}^{2m} \left(\frac{1}{\ln 2} A\right) dx\hat{a}_x \times \mu_o \frac{1A}{2\pi x}\hat{a}_y$$

$$= \left(\frac{1}{\ln 2}A\right)(1A)\frac{\mu_o}{2\pi}\int_{1m}^{2m}\frac{dx}{x}\hat{a}_z = \left(\frac{1}{\ln 2}A\right)(1A)\frac{\mu_o}{2\pi}\ln x\Big|_{1m}^{2m}\hat{a}_z$$

$$= \left(\frac{1}{\ln 2}A\right)(1A)\frac{\mu_o}{2\pi}\ln(2) = \frac{4 \times 10^{-7}\frac{H}{m}}{2\pi}A^2\hat{a}_z$$

$$= 2 \times 10^{-7}\hat{a}_z \frac{H}{m}A^2$$

$$\left[\frac{H}{m}A^2 = \frac{Wb}{Am}A^2 = \frac{VsA}{m} = \frac{JsC}{Cms} = \frac{J}{m} = N\right]$$

$$= 2 \times 10^{-7}\hat{a}_z \, N$$

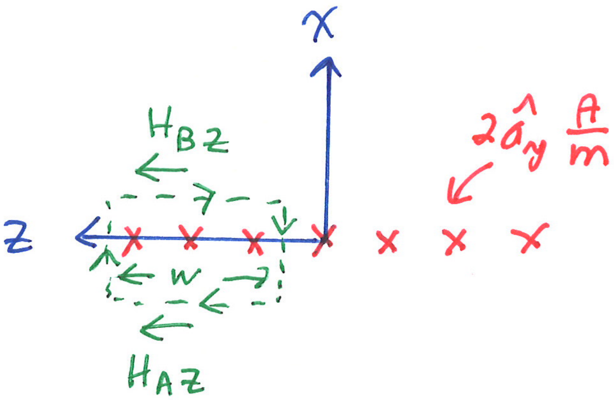
(12 pts) 4. The region x < 0 has $\mu = \mu_o$ and $\mathbf{H} = 4\hat{\mathbf{a}}_x + 6\hat{\mathbf{a}}_y - 4\hat{\mathbf{a}}_z \frac{A}{m}$. The region x > 0 has permeability $\mu = 2\mu_o$. There is a sheet current density flowing on the plane x = 0 of $\mathbf{K} = (2\hat{\mathbf{a}}_y + 2\hat{\mathbf{a}}_z)\frac{A}{m}$. Find the magnetic field intensity for x > 0.

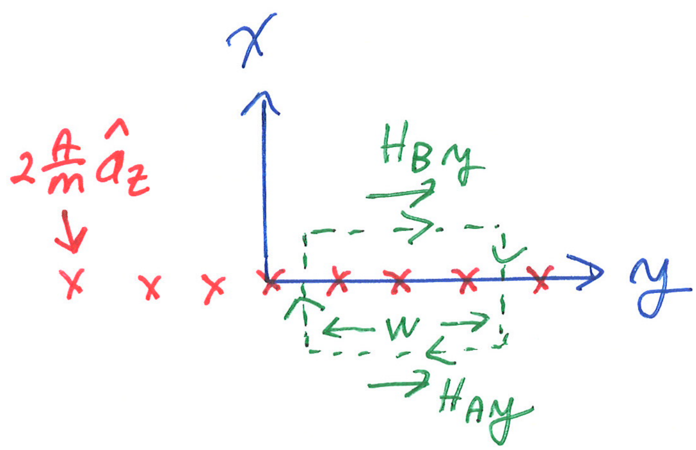
let x<0 be region A and x>0 region B

$H_A = 4\hat{a}_x + 6\hat{a}_y - 4\hat{a}_z \frac{A}{m} \quad \mu_A = \mu_o \quad \mu_B = 2\mu_o$

$B_{AN} = \mu_o H_{AN} = B_{BN} = 2\mu_o H_{BN}$

$H_{BN} = \frac{\mu_o}{2\mu_o} H_{AN} = \frac{1}{2} H_{Ax} = \frac{1}{2}\left(4\frac{A}{m}\right) = 2\frac{A}{m} = H_{Bx}$





$-H_{Bz}W + H_{Az}W = \left(2\frac{A}{m}\right)w$

$H_{Bz} = H_{Az} - 2\frac{A}{m}$

$H_{Bz} = -4\frac{A}{m} - 2\frac{A}{m}$

$H_{Bz} = -6\frac{A}{m}$

$H_{By}W - H_{Ay}W = \left(2\frac{A}{m}\right)w$

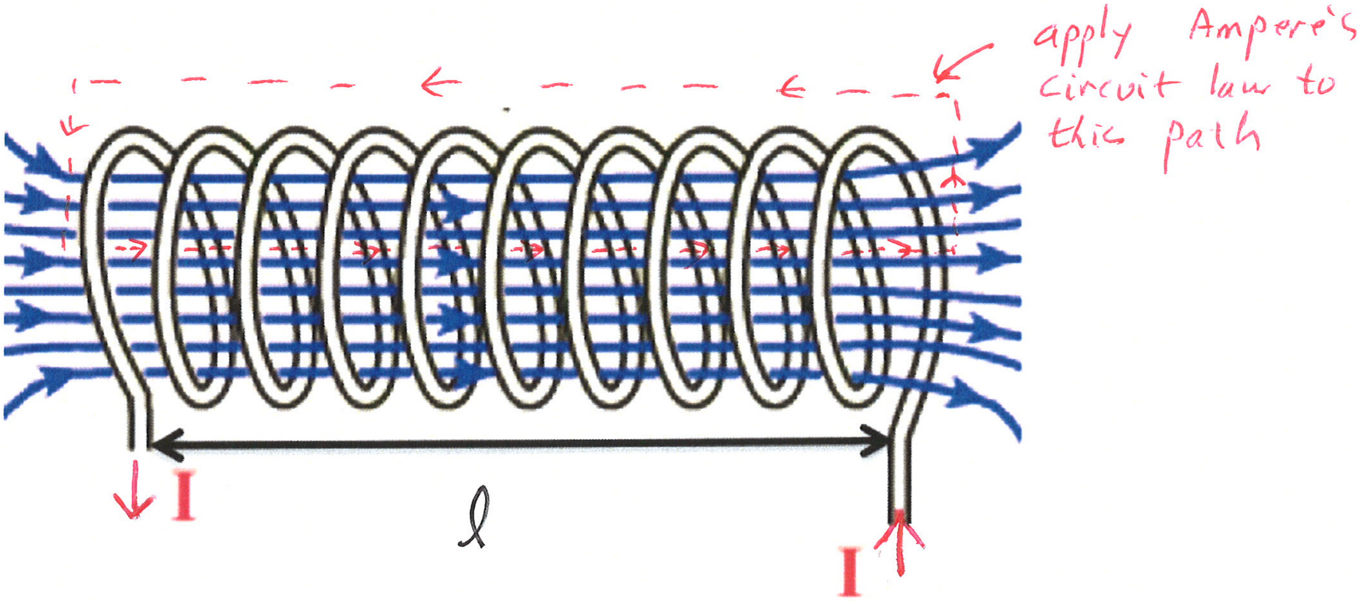
$H_{By} = H_{Ay} + 2\frac{A}{m}$

$H_{By} = 6\frac{A}{m} + 2\frac{A}{m}$

$H_{By} = 8\frac{A}{m}$

$$\vec{H}_A = \left(2\hat{a}_x + 8\hat{a}_y - 6\hat{a}_z\right)\frac{A}{m}$$

(10 pts) 5. Find the inductance of a tightly wound solenoid of N turns, length l, and radius a. You need to show the steps of deriving the equation for the inductance.



$$\oint H \cdot d\ell = I_{enclosed} = NI$$

The H-field is weak outside the solenoid so $\int \vec{H} \cdot d\vec{\ell}$ outside the solenoid is negligible.

$$H\ell = NI$$

$$H = \frac{NI}{\ell} \implies B = \mu_0 H = \frac{\mu_0 NI}{\ell}$$

The cross-sectional area is $S = \pi a^2$

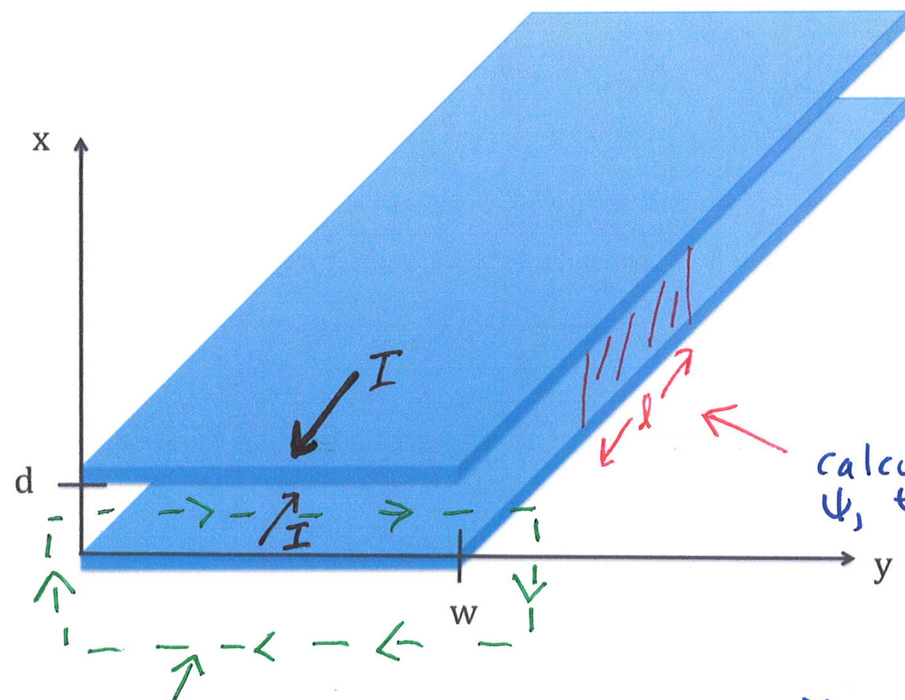
so $\Psi = BS = \dfrac{\mu_0 N I \pi a^2}{\ell}$

$$\lambda = N\Psi = \frac{\mu_0 N^2 I \pi a^2}{\ell}$$

$$L = \frac{\lambda}{I} = \frac{\mu_0 N^2 \pi a^2}{\ell}$$

(10 pts) 6. Determine the inductance per unit length for the parallel plate transmission line shown. Assume the transmission line is of infinite length in the z direction and ignore fringing fields. The plate spacing is d and w >> d. The plates are parallel to the xy-plane and the current flow is in the $-\hat{\mathbf{a}}_z$ direction on the top plate and the $+\hat{\mathbf{a}}_z$ direction on the bottom plate.

This is similar to the solenoid we discussed in class in the the H-field is very weak outside the plates and can be ignored compared to the field between the plates



calculate magnetic flux, Ψ, through this cross-section

$$\oint \vec{H} \cdot d\vec{l} = H_{inside} w = I_{enclosed} = I$$

$$\vec{H}_{inside} = \frac{I}{w} \hat{a}_y$$

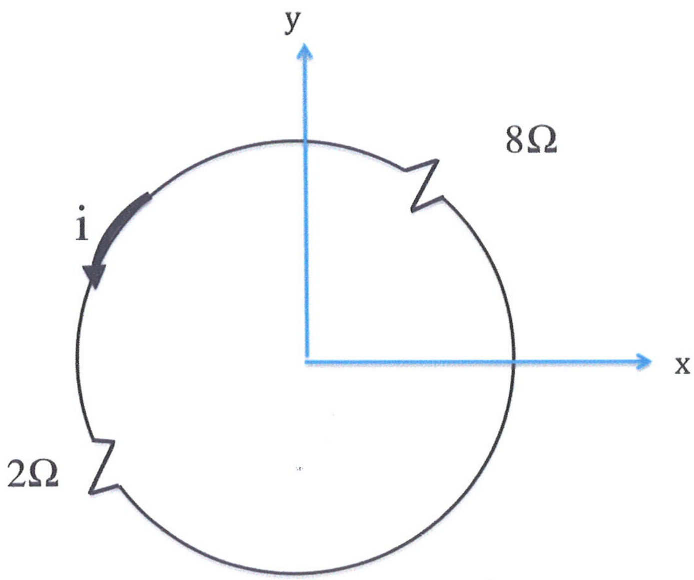
$$B_{inside} = \mu_o \frac{I}{w} \hat{a}_y$$

$$\Psi = B_{inside} \, l d = \mu_o \frac{I}{w} l d$$

$$L = \frac{\Psi}{I} = \frac{\mu_o l d}{w}$$

$$\frac{L}{l} = \frac{\mu_o d}{w} = \text{inductance per unit length}$$

(10 pts). 7. Shown are two resistors connected by wires of negligible resistance. This circuit is in the xy-plane. If the area of the loop is $2m^2$, if everywhere the magnetic flux density is $B = 10\sin(10t)\hat{a}_z T$ and with the current defined as shown, find the current flowing in the loop



$$\oint \vec{E}\cdot\vec{dl} = -\frac{d\psi}{dt}$$

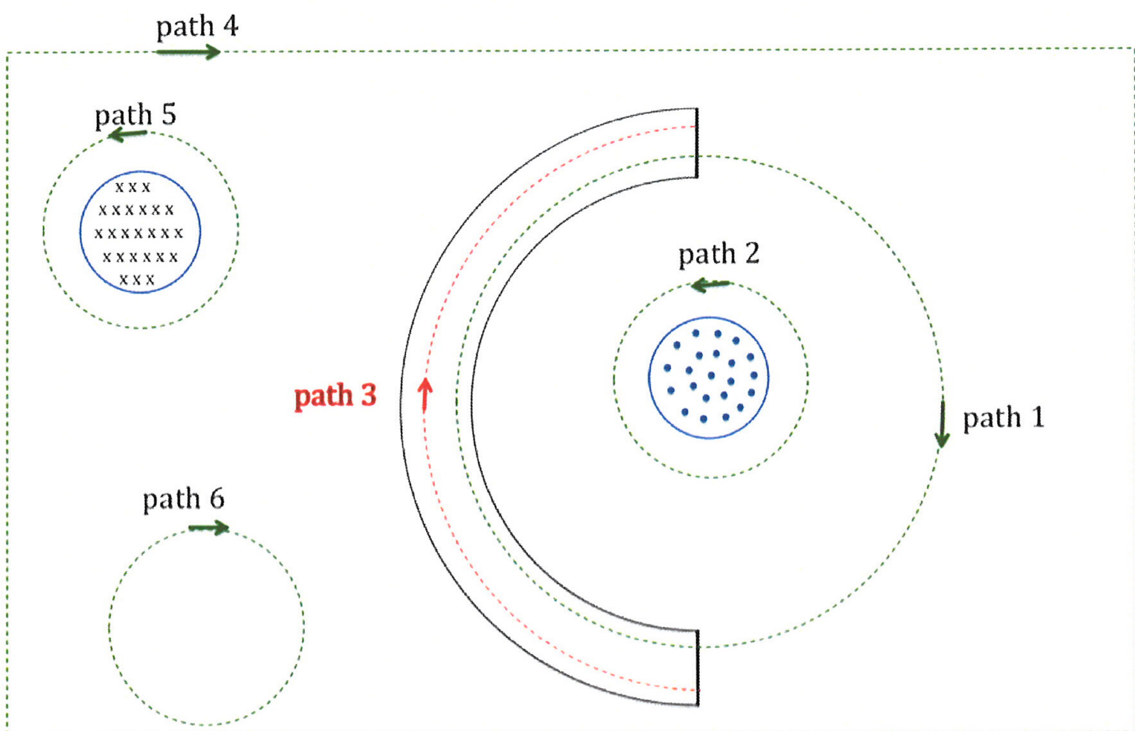
$$(2\Omega)i + (8\Omega)i = -\frac{d}{dt}(BS) = -\frac{d}{dt}\left[10\sin 10t\right]2\ Tm^2$$

$$i = -2\frac{d}{dt}\left[\sin 10t\right]\frac{Tm^2}{\Omega} = -2\frac{d}{dt}\left[\sin 10t\right]\frac{Wb}{\Omega}$$

$$i = -20\cos 10t\ \frac{Wb}{s\Omega} = -20\cos 10t\ \frac{Vs}{s\Omega}$$

$$i = -20\cos 10t\ A$$

(15 pts) 8. Shown are the cross-sections of two infinitely long solenoids. The one with the dots indicates a $\frac{d\psi}{dt}=10V$ out of the page. The one with the x's indicates a $\frac{d\psi}{dt}=10V$ in to the page. The half ring has a total resistance of 2 Ω from one end of path 3 to the other. Find the values of $\oint \mathbf{E} \cdot \mathbf{dl}$ over the paths indicated.



The $\oint \mathbf{E} \cdot \mathbf{dl}$ over path 1 = 10 V

The $\oint \mathbf{E} \cdot \mathbf{dl}$ over path 4 = 0

The $\oint \mathbf{E} \cdot \mathbf{dl}$ over path 2 = -10V

The $\oint \mathbf{E} \cdot \mathbf{dl}$ over path 5 = 10 V

The $\int \mathbf{E} \cdot \mathbf{dl}$ over path 3 = 0

The $\oint \mathbf{E} \cdot \mathbf{dl}$ over path 6 = 0

(3 pts) What current is flowing along path 3?

0

(16 pts) 9. Circle true or false concerning the statements for a ferromagnetic material.

The integral, $\oint \mathbf{H} \cdot d\mathbf{S}$, over a closed surface will always be zero.	True	False
The integral, $\oint \mathbf{E} \cdot d\mathbf{l}$, over a closed path will always be zero.	True	False
The net force on a current loop in a uniform magnetic flux density field will be zero.	True	False
At the Curie temperature the north and south poles of a bar magnet will change positions.	True	False
For a ferromagnetic material, when all the magnetic domains are pointing in the direction of the external magnetic field, the slope of the H versus B curve will be μ_o	True	False
A diamagnetic material is repelled by a permanent magnet.	True	False
A ferromagnetic material will retain a considerable amount of magnetization when removed from an external magnetic field.	True	False
The force on a moving charge in a magnetic field does no work.	True	False